Bias-free Nonparametric Estimation of Intra-Day Trade Activity Measures

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ABSTRACT

We propose a bias-free method to estimate intra-day activity measures along the lines of Gouriéroux, Jasiak, and Le Fol (1999). In an empirical illustration using NYSE tick-by-tick data we find that standard fixed bandwidth kernel smoothers lead to severely biased estimators of intra-day activity measures. As a consequence, investment and trading strategies that are based on these standard methods can be misleading. We show that by employing recently proposed kernel methods the estimation of intra-day activity measures can be performed in a superior way.

Keywords: Financial transactions data, liquidity, trading intensity, non-parametric methods, beta kernel, boundary bias.

JEL classification: C14, C41, D4.

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1. INTRODUCTION

The notion of liquidity is a crucial one in economics and finance. While there is probably little doubt about the importance of this concept for any theory of asset allocation, at the same time there is no operational, generally accepted definition that indicates how to *measure* the degree of an asset's liquidity. As Grossman and Miller (1988) put it: "*The T-bond Futures pit at the Chicago Board of Trade is surely more liquid than the local market for residential housing. But how much more*?"

Broadly speaking, liquidity can be considered as an asset's ability to be traded in large *quantities* at reasonable *prices* given the demand and supply conditions at the time of the trade. While this definition is certainly intuitively appealing, it does not provide us with an unambiguous measure of liquidity that could be used e.g. for comparison between assets or markets. Yet it emphasizes that volume, price and time to (order) execution are the principal factors that contribute to the liquidity of an asset. Commonly used measures of liquidity such as the bid-ask spread, the liquidity ratio or the variance ratio fail to account for these three factors appropriately and none of these liquidity indicators explicitly incorporates the time factor.[1] This drawback is addressed by Gouriéroux, Jasiak, and Le Fol (1999), henceforth referred to as GJL, who introduced a new class of intra-day market liquidity measures. By distinguishing different trading events GJL are able to account for all three liquidity components. These measures are built on the trade arrival process in the first place, but can be easily modified to take account of other characteristics of the trade. Extensions of this basic approach focus

[1] For a general critique of these liquidity measures see Grossman and Miller (1988) and Schwartz (1992).

on the time until a pre-specified volume or value has been traded or until a pre-specified price change has occurred.

GJL show how their indicators can be used for a variety of comparative financial analyses, such as assessing investment strategies based on liquidity orderings between assets or parallel markets and propose that non-parametric estimation techniques be employed. More precisely, a Gaussian kernel function with a fixed bandwidth parameter is used to estimate intra-daily intensity, density and survivor functions. In this paper we will argue that although the GJL approach towards measuring liquidity is an innovative and promising one, the use of the standard kernel methods implies a major drawback. We will argue that it leads to severely biased liquidity measures near the opening and the close of the trading day. Adopting a variable kernel estimator for density functions recently proposed by Chen (1999a) we will present a simple and effective method to estimate intra day trade intensity functions that avoids these drawbacks. Measures of market liquidity are based on these estimates.

In the empirical section we will demonstrate the practical relevance of our alternative approach. We focus on measuring intra day intensity functions of selected stocks traded at the NYSE. It is a well known stylized fact that trading intensities at the NYSE evolve during the trading day according to a U-shaped pattern, i.e. it shows peaks at the opening and the close.[2] Using GJL's preferred method, however, estimates of the intensity function take on an implausible M-shape that is induced by the use of a fixed-bandwidth

[2]These patterns have been recognized in recent studies of inter transaction duration dynamics using the ACD-framework [Engle and Russell (1998), Engle (2000)]. Similar U-shaped patterns have been found for other variables, including volume, volatility and the bid-ask spread in numerous studies of NYSE trade data sets. For a summary of these findings see Goodhart and O'Hara (1997).

kernel. Applying our alternative approach, the expected shape of the NYSE intra-day intensity function can be recovered for all stocks.

This paper is organized as follows. In Section 2 we summarize the main statistical properties of the trading activity model used by GJL. Section 3 briefly describes the basics of non-parametric estimation of intensity functions. In section 4 we discuss the sources of the boundary bias inherent in GJL estimation techniques. We exemplify the severity of the bias with a small scale simulation study and propose an essentially unbiased remedy that is easy to implement. An empirical illustration of our approach is given in section 5, where we use data for a selection of 5 stocks traded on the NYSE in 1996 to estimate intra day trade intensity functions. Section 6 concludes the paper.

2. A STATISTICAL MODEL OF INTRA-DAY MARKET ACTIVITY

Consider the process of trading an asset on an exchange during a single day. In accordance with GJL we assume that trade arrivals evolve randomly in time according to a time dependent Poisson process with intensity function $\lambda(t)$ that depends on the clock time of day. The counting process $N(t)$ gives the number of trades observed from the beginning of the trading day until t. Trades occur according to the following probability law:

$$P[N(t+dt) - N(t) = 0] \; = \; 1 - \lambda(t) \cdot dt + o(dt), \tag{1}$$

$$P[N(t+dt) - N(t) = 1] \; = \; \lambda(t) \cdot dt + o(dt), \tag{2}$$

which implies

$$P[N(t+dt) - N(t) > 1] \; = \; o(dt). \tag{3}$$

Thus, the probability of observing a trade in the interval $(t, t+dt)$ depends on the length of the interval dt and on the intensity or hazard function $\lambda(t)$, which gives the instantaneous rate of trading per unit of time. The expression $o(dt)$ denotes any function of dt such that $\lim\limits_{dt \to 0} \frac{o(dt)}{dt} = 0$.

Based on the theory of self-exciting point processes that dates back to Cox and Lewis (1966), Hawkes (1971a, 1971b, 1972) and Rubin (1972), Engle and Russell (1998) define an intensity rate conditional on the complete history of the trading process since the beginning of the day at t^{min},

$$\lambda(t \mid \mathcal{F}(t)) = \lim_{dt \to 0} \frac{P[N(t+dt) - N(t) > 0 \mid \mathcal{F}(t)]}{dt}, \tag{4}$$

where $\mathcal{F}(t) = \{N(t), t_1, \ldots, t_{N(t)}\}$. GJL restrict the conditioning information set so that it contains only the time of day, $\mathcal{F}(t) = t$, which yields[3]

$$\lambda(t) = \lim_{dt \to 0} \frac{P[N(t+dt) - N(t) > 0 \mid t]}{dt}. \tag{5}$$

The integrated intensity function $\Lambda(t) = \int_{t^{min}}^{t} \lambda(u) du$ is equal to the expected number of trades from the opening until t, $\Lambda(t) = E[N(t)]$. GJL show how liquidity orderings between one asset at different times (or traded at different markets) as well as orderings between two different assets may be conducted using estimates of the survivor function conditional on the time of day. These liquidity measures are based on the volume and value processes rather than the pure trading process introduced in this section. In order to keep our exposition simple, we will concentrate on the estimation of the intensity function for the trading process, which itself can be used as a liquidity measure in the case of an investor whose only concern is the ability to trade a unit share quickly. Since the intensity rate $\lambda(t)$ is proportional to the probability of

[3]This rules out any dependency of the intensity rate on quantities other than the time of day. Engle and Russell (1998) and Hamilton and Jorda (1999) have introduced parametric models that allow more general intensity rate dynamics.

trading in the next instant of time, conditional on the time of day, an asset a can be considered as being more liquid at time t than asset b if and only if the following relation holds

$$
\begin{aligned}
(6) \qquad & P_a[N(t+dt) - N(t) = 1] \;>\; P_b[N(t+dt) - N(t) = 1] \\
(7) \qquad & \Leftrightarrow \lambda_a(t) \;>\; \lambda_b(t).
\end{aligned}
$$

Analogous orderings can be conducted using estimates of the integrated intensity function $\Lambda(t)$ and these can be based on any of the associated volume, value or even price change processes.

3. NON-PARAMETRIC ESTIMATION OF THE INTENSITY RATE

A consistent non-parametric estimator of the intensity function (5) has been proposed by Ramlau-Hansen (1983), extending an earlier contribution by Watson and Leadbetter (1964).[4] The basic idea is to conduct a kernel smoothing of the observed occurrence rates $\frac{1}{Y(t_n)}$ where $Y(t_n)$ is the number of statistical units that are exposed to the risk of going through a transition at the time t_n,

$$
(8) \qquad \hat{\lambda}(t) = \frac{1}{h} \cdot \sum_{n=1}^{N} K\left(\frac{t - t_n}{h}\right) \cdot \frac{1}{Y(t_n)},
$$

h is a bandwidth parameter and $K(\cdot)$ an appropriate kernel function. $t_1 < t_2 < \ldots < t_N$ denotes the sequence of jump times of the underlying count process. In the context of this paper this refers to the time of day at which an asset is being traded. Since we restrict our attention to a single asset we always have $Y(t_n) = 1$.

The estimator $\hat{\lambda}(t)$ is a weighted average of the observed increments of the count process over the range $[t - h, t + h]$, i.e. only observations in this

[4]See also Andersen, Borgan, Gill, and Keiding (1992), ch. IV.2.1.

interval contribute to the sum. This holds true for all symmetric kernels except the Gaussian, which is usually not bounded, but gives negligible weights to observations that are more than $4h$ away from t.

GJL propose to treat each trading day in the sample as a distinct, independent realization of the counting process and thus to estimate the intensity rate as an average of the Ramlau-Hansen estimators for separate days. Denote the n-th observed trading time on day m as $t_n(m)$. For a total of M trading days we have

$$\hat{\lambda}(t) = \frac{1}{M} \cdot \sum_{m=1}^{M} \left[\frac{1}{h} \cdot \sum_{n=1}^{N_m} K\left(\frac{t - t_n(m)}{h} \right) \right]. \tag{9}$$

4. TACKLING THE BOUNDARY BIAS PROBLEM

The boundary bias arises as a consequence of using a fixed bandwidth h together with a symmetric kernel function $K(\cdot)$ to estimate densities and related quantities, such as the intensity function, with compact support. For example, the widely used Epanechnikov kernel is defined as

$$K_E(u) = 0.75 \cdot \left(1 - u^2\right) \cdot I\left(|u| \leq 1\right). \tag{10}$$

This kernel assigns positive weights to any value of $u \equiv \left(\frac{t-t_n(m)}{h}\right)$ that is smaller than one in absolute value, while all $u > 1$ receive a weight of zero. Since the trading times always lie in a closed interval $[t^{min}, t^{max}]$, the Epanechnikov kernel implicitly gives weight to values of u *outside* the admissible range in a neighborhood of the bounds. Because there are no observations outside the range, observations in the neighborhood of the bounds will receive weights that are too small. Hence, the amount of smoothing will be systematically higher than during the rest of the day. As a consequence, estimates of the intensity function based on (9) will be biased towards zero in any of the boundary regions given by $[t^{min}, t^{min}+h]$ and $[t^{max}-h, t^{max}]$. To illustrate the significance of the boundary bias problem we simulate a sequence of 600 equally spaced trading times. The simulation mimics a 6.5 hour NYSE trading day which starts at 9.30 a.m. and ends at 4 p.m. Based on the simulated data, we estimate the conditional intensity function according to (9). In addition to the Epanechnikov kernel (10) we employ three other standard kernel functions, namely the Gaussian $K_G(u)$, uniform $K_U(u)$ and the quartic $K_Q(u)$,

$$K_G(u) = \frac{1}{\sqrt{2\pi}} \cdot \exp\left(-0.5u^2\right), \tag{11}$$

$$K_U(u) = 0.5 \cdot I\left(|u| \leq 1\right), \tag{12}$$

$$K_Q(u) = \frac{15}{16}\left(1-u^2\right)^2 \cdot I\left(|u| \leq 1\right). \tag{13}$$

We restrict our attention to these four kernels in order to keep the exposition brief. We have also employed triangular, triweight and cosinus kernels, but the results were not qualitatively different. The bandwidth h varies between 10 and 35 minutes in order to exemplify the influence of bandwidth choice on the boundary bias. This choice is motivated by GJL who use a Gaussian kernel with bandwidth parameter equal to 22 minutes.

The intensity function for the simulated data should be a straight line with a value of 0.0257, which is equal to the inverse of the mean duration of 38.94 seconds. As figure 1 shows, all kernel functions do a satisfactory job during the central part of the trading day, regardless of the choice of kernel and bandwidth. However, a strong downward bias is clearly visible near the opening and close. This implies that the trading intensity is severely underestimated. Furthermore, choosing a larger value for the bandwidth, i.e. applying more smoothing to the data, makes things worse. From our exposition of the boundary bias problem, this phenomenon is not at all un-

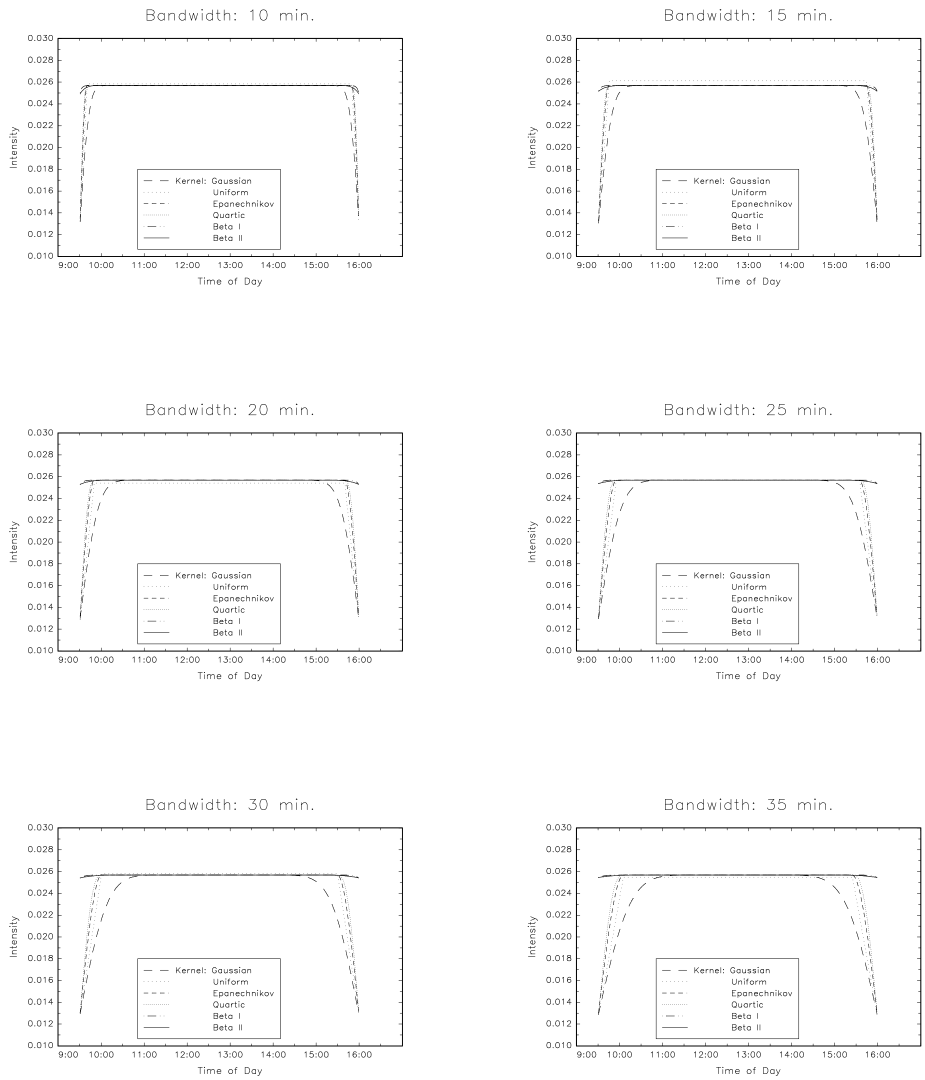

FIGURE 1: Estimated intensity rates for 600 equally spaced trading times. The bandwidth parameter varies from 10 minutes (upper left panel) to 35 minutes (lower right panel).

expected. Note that the Gaussian is clearly the kernel that is most affected by the boundary bias. This is also an expected result, since the Gaussian is the only kernel with an unbounded domain.

The problem of boundary bias for densities with compact support has long been recognized in the literature. Silverman (1986) discusses several ad hoc bias corrections. Recently, the statistics literature exhibited renewed interest in the boundary bias problem (see, among others Müller, 1991, Marron and Ruppert, 1994, Cheng, Fan, and Marron, 1996, Jones and Foster, 1996). Chen (1999a) proposed using the density function of the beta distribution as a kernel smoother for data with bounded support. Let the trading times have compact support on the interval (t^{min}, t^{max}) and denote standardized trading times by $z_n(m) \equiv \frac{t_n(m)-t^{min}}{t^{max}-t^{min}}$. The beta type-I estimator for the intensity function $\lambda(t)$ is given by

$$(14) \quad \hat{\lambda}_I(t) = \frac{1}{M} \cdot \sum_{m=1}^{M} \left[\frac{1}{N_m \cdot (t^{max} - t^{min})} \cdot \sum_{n=1}^{N_m} B_{\frac{z}{h}+1, \frac{1-z}{h}+1} \left(z_n(m) \right) \right],$$

where h is a smoothing parameter satisfying $0 \leq h \leq 1$. $B_{p,q}(z)$ denotes the density of a standard beta random variable $z \in (0,1)$ with parameters p and q, i.e.

$$(15) \qquad B_{p,q}(z) = \frac{\Gamma(p+q)}{\Gamma(p) \cdot \Gamma(q)} \cdot z^{p-1} \cdot (1-z)^{q-1},$$

and $\Gamma(\cdot)$ denotes the Gamma function. Chen (1999a) also proposes a second version of the beta kernel density estimator that is designed to reduce bias in finite samples,

$$(16) \qquad \hat{\lambda}_{II}(t) = \frac{1}{M} \cdot \sum_{m=1}^{M} \left[\frac{1}{N_m \cdot (t^{max} - t^{min})} \cdot \sum_{n=1}^{N_m} B^*_{z,h} \left(z_n(m) \right) \right],$$

where $B^*_{z,h}(.)$ is a boundary adapted beta kernel defined as

$$B^*_{z,h}(z) = \begin{cases} B_{\rho(z,h),\frac{1-z}{h}} & \text{if} \quad z_n(m) \in [0, 2h) \\ B_{\frac{z}{h},\frac{1-z}{h}} & \text{if} \quad z_n(m) \in [2h, 1-2h] \\ B_{\frac{z}{h},\rho(1-z,h)} & \text{if} \quad z_n(m) \in (1-2h, 1] \end{cases}, \tag{17}$$

and

$$\rho(z,h) = 2.5 + 2h^2 - \sqrt{2.25 + 4h^4 + 6h^2 - z^2 - \frac{z}{h}}. \tag{18}$$

The beta kernel possesses a number of advantages compared with the alternatives, as it is always nonnegative and it matches the support of the true density by construction. The effective sample size used in the estimation is equal to the total sample, which leads to a lower finite sample variance. Because of the flexible shape of the beta distribution, the amount of smoothing is automatically altered as the trading times approach one of the boundary regions without explicitly altering the value of the bandwidth parameter.

We employed both variants of the beta kernel estimator to the simulated data. Figure 1 contains the implied intensity functions. It is obvious that both beta kernel estimators are able to remove the boundary bias almost completely. Only in a very small region near the boundary can a negligible downward shift be detected. In contrast to the behaviour of all other kernels, an increase of the bandwidth leads to a reduction of the remaining bias of the beta kernel. Based on an informal inspection of this small scale simulation, there seems to be little to choose between the two versions of the beta kernel. However, as both asymptotic reasoning and monte carlo evidence presented in Chen (1999a) reveal, the beta type II estimator is preferable as it has both a smaller bias and smaller variance than the beta type I.

5. EMPIRICAL ILLUSTRATION

Our data set consists of a sample of 5 stocks traded on the NYSE from 6/3/96 until 12/31/96 from the TAQ (Trades And Quotes) data set. We use all shares traded during the regular trading interval (9:30 a.m. until 4:00 p.m.) for the following companies: Disney (DIS), Coca Cola (KO), IBM, Exxon (XON) and Boeing (BA). The data set contains information on the timing of the trades, prices, volume, returns, spreads as well as every revision of best bid and ask prices and corresponding volumes during the trading day.

For our empirical illustration, consecutive trades that were recorded with a duration equal to zero were aggregated. In a preliminary step we divide the trading day in 39 equally spaced 10 min. sub-intervals. Figure 2 displays the mean duration of all spells beginning in a particular interval.

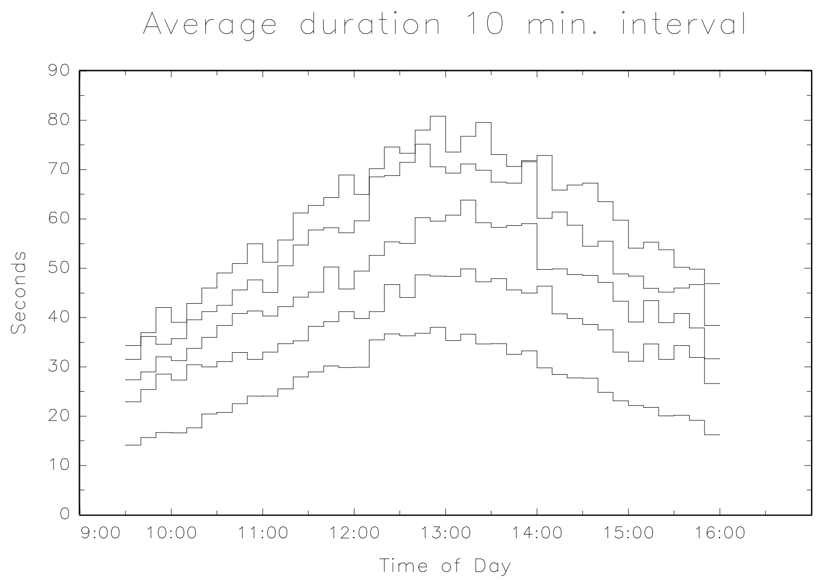

FIGURE 2: Average intertrade durations for selected shares traded at the NYSE between 6/3/96 and 12/31/96. From top to bottom: Boeing, Exxon, Disney, Coca Cola, IBM.

Counting the number of trades $n_{i,m}$ in interval i on day m, a simple alternative estimate of the intensity function can be obtained by

$$\hat{\lambda}(i) = \frac{1}{M} \cdot \frac{\sum_{m=1}^{M} n_{i,m}}{\Delta t}, \tag{19}$$

where Δt is the predetermined length of the interval (Cox and Lewis, 1966).

Compared with the non-parametric estimators discussed in section 3 and 4, (19) implies a number of drawbacks. First, it is less efficient, since only the counts in the fixed interval are taken into account for the estimation of $\lambda(i)$. Second, the fact that (19) is a step-function is inconsistent with the continuity of the trading process. Third, the estimator is sensitive toward

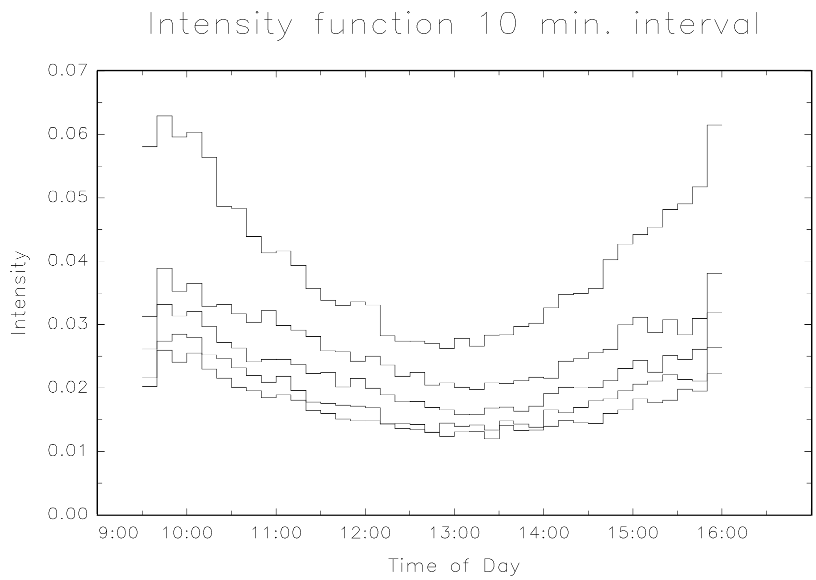

FIGURE 3: Estimates of the intensity function based on the counting process for selected shares traded at the NYSE between 6/3/96 and 12/31/96. From top to bottom: IBM, Coca Cola, Disney, Exxon, Boeing.

local variations caused by random noise contained in the data. Compared to the fixed-bandwidth kernel smoothers, however, it has the advantage that it is robust against the boundary bias. Hence, it provides a crude, yet unbiased approximation to the shape of the true intensity function. Figure 3 depicts the intensity functions estimated using (19). Both figure 2 and figure 3 confirm that the trading intensity at the NYSE is roughly U-shaped, which is in line with the evidence in Engle and Russell (1998) and Engle (2000). The deviations from the stylized U-shape that are indicated close to the opening are caused by the NYSE opening auction. Figures 4 and 5 depict the

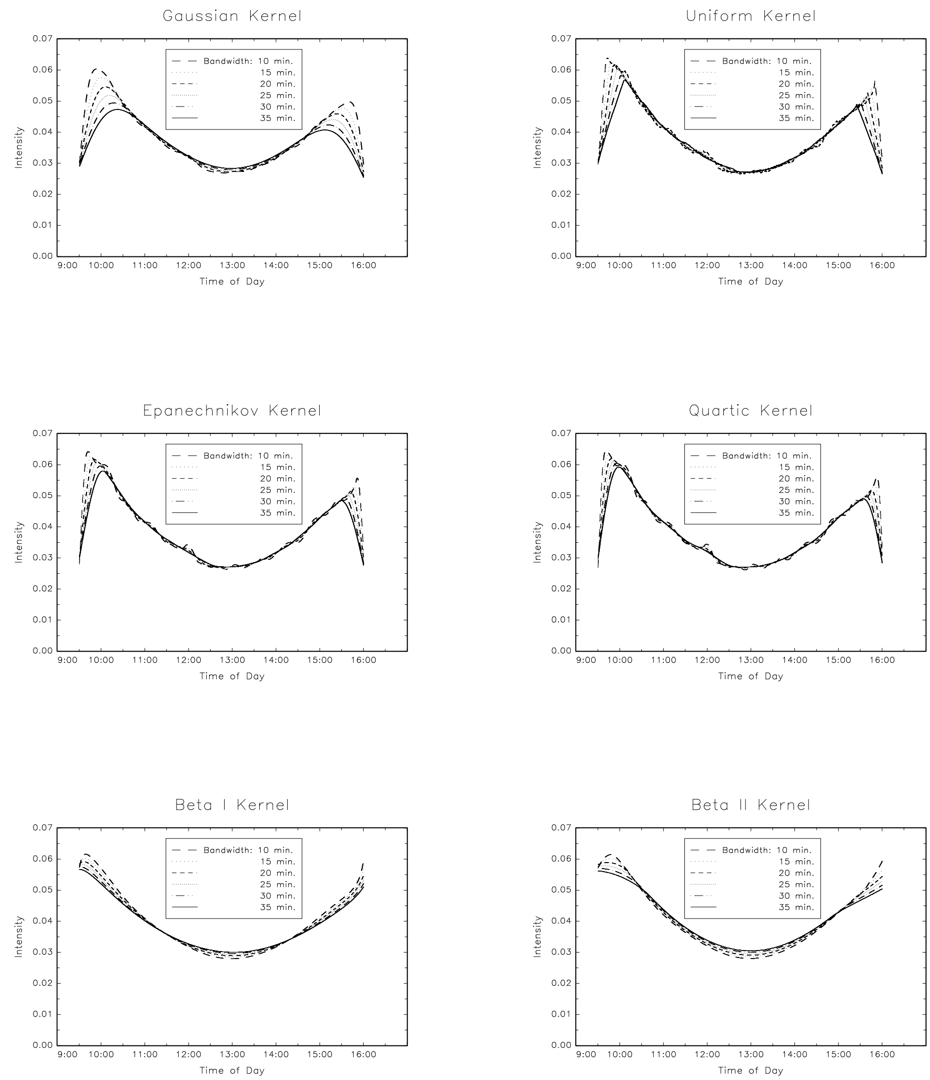

FIGURE 4: Estimated intensity rates for IBM share transactions at the NYSE between 6/3/96 and 12/31/96 (138.324 observations).

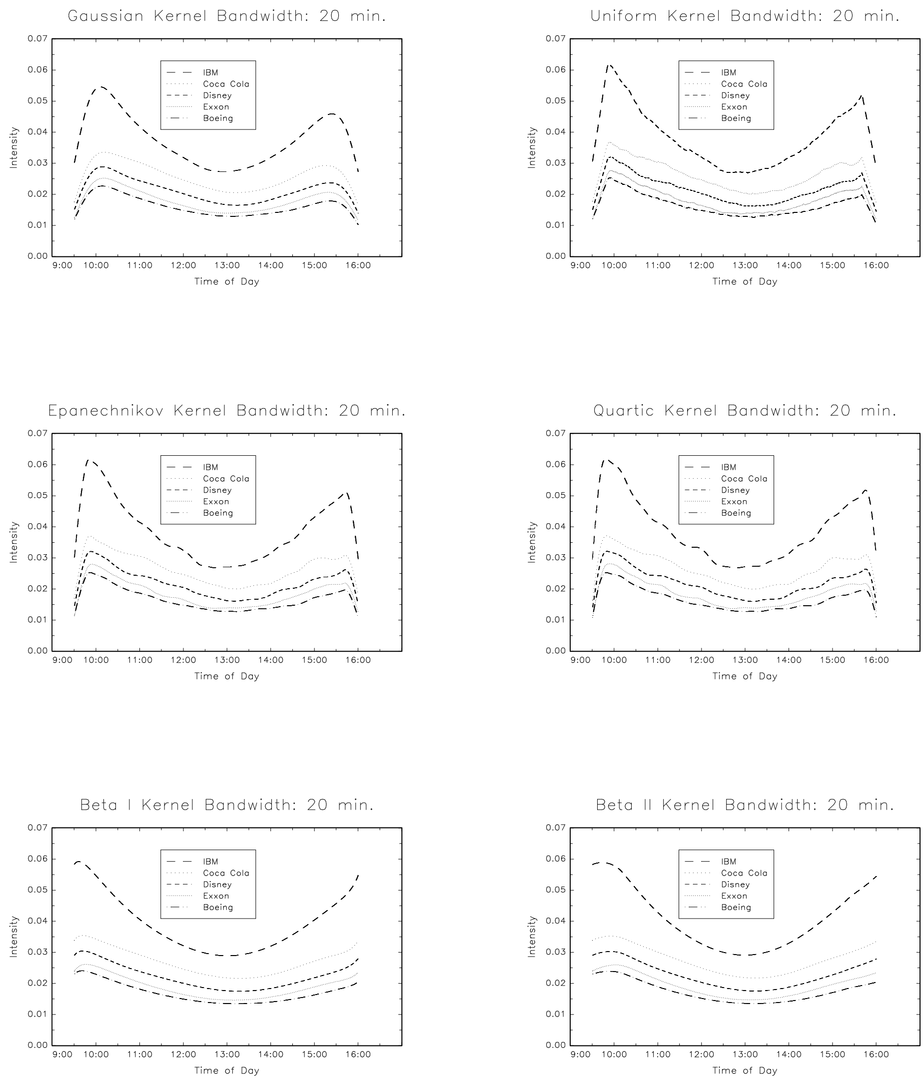

FIGURE 5: Estimated intensity rates for selected shares traded at the NYSE between 6/3/96 and 12/31/96.

intensity function estimates employing the four standard kernel estimators and both versions of the beta kernel. The estimates for the most actively traded share, IBM, for various choices of the bandwidth parameter are given in figure 4. Regardless of the chosen bandwidth, the standard kernel estimators produce an unmistakable M-shape of the intensity function that is even more pronounced with increasing bandwidth. This implausible result is obviously an effect of the inherent boundary bias of the fixed bandwidth kernel. By contrast, the two beta kernel estimators do a good job of recovering the expected U-shape of the NYSE intensity function. This is confirmed by the picture given in figure 5, where we reproduced estimates for all shares using a fixed bandwidth of 20 minutes.

The boundary bias works like a disguise, hiding a number of interesting features of the trading process near the opening and close: First, the non-monotonic shape of the intensity function at the opening, which is detectable using smaller bandwidth beta kernel estimators, is not an artefact caused by the boundary bias, but an economically plausible effect of the opening auction. Second, the apparent decrease of trading intensity at the close suggested by the fixed-bandwidth kernel estimators is solely due to the boundary bias: The boundary bias-free beta kernel estimators clearly reveal an *increasing* trading intensity at the close. Thus, the use of the beta kernel estimator discloses an asymmetry of NYSE trading behaviour between the opening and the close that is not detectable by standard methods.

6. CONCLUSIONS

A meaningful measure of liquidity provides an important input for an investor's asset allocation and trading strategies. Yet there is no operational, generally accepted definition that indicates how to measure the degree of an asset's liquidity. Recognizing that liquidity can be interpreted as the ability to trade an asset quickly in large quantities at reasonable prices, Gouriéroux, Jasiak, and Le Fol (1999) have introduced a new class of intra-day market activity measures that are designed to account for these three components of liquidity.

In this paper we have argued that despite the originality and practical relevance of their approach, the standard non-parametric estimators that have been employed in GJL's seminal paper suffer from a non-negligible deficiency. We have shown in a simulation study and using NYSE trade data that the standard methods employed by GJL produce biased estimates of the intra-day intensity functions near the opening and the close of the trading day. We have proposed a straightforwardly implementable method that circumvents this problem and showed that interesting features of the NYSE trading process, that are disguised when the standard methods are applied, can be recovered using our alternative estimators.

In this paper we have focused on one of several possible liquidity measures that can be constructed within the GJL framework. However, generalizations of our approach with respect to other liquidity measures are straightforward. Other quantities that have been used by GJL to describe the trading process in more detail include non-parametric estimators of the density and the survivor function for volume and turnover durations, which are affected by the same shortcomings that we discussed in this paper. The estimate of the density function of the volume or turnover durations conditional on the time of day requires a second kernel smoothing of the inter trade durations, which have a non-negative support. A superior density estimator could therefore combine two recently proposed kernel smoothers, the beta kernel for the trading times data discussed in this paper, and the gamma kernel proposed by Chen (1999b) for the duration data.

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